Estimating Probability Distribution of Benefit from Flood Control Projects

Abstract—Frequencies of severe flood events are expected to increase due to a changing climate. Therefore, it becomes more and more important to implement suitable flood defenses based on quantitative benefit analysis of flood control projects such as dam construction and embankment. In general, the benefit from flood control projects is defined as the sum of expected value of annual reduced damage cost over the evaluation years, considering discount rate of each year. Since flooding is low-frequency events especially in highly protected areas, it is highly uncertain whether actual reduced damage cost is distributed around its expected value, i.e. benefit calculated from the expected value does not necessarily occur frequently. The probability distribution of reduced damage cost would provide more comprehensive information for decision making. This study redefined benefit from a flood control project as a reduced damage cost, which is actually a random variable, and developed its probability distribution by applying an extreme rainfall generation method. Then, we demonstrated the presented method under several scenarios of dam construction projects. The estimated probability distribution was skewed leftwards, and had a long tail. It reveals large variability of reduced damage cost. From the probability distribution, we can extract several statistics in addition to the expected value, which provide more information to discuss the characteristics of actual reduced damage cost by flood control projects.

Keywords— benefit from flood control projects; probability distribution; uncertainty;

I. INTRODUCTION

It is concerned that frequencies of intenserainfall events will increase due to climate change. To implement adaptation measures against the expected severe flood events,countermeasuresshould be taken, and fundamental measures are constructing flood defensessuch as embankment and dam.In Japan, cost-benefit analysis is utilized from 1997 to evaluate the efficiency of public works, where benefit corresponds toreduced inundation damage cost by installing flood defenses, and the efficiency of flood control projects are measured by comparing it with the cost for construction and maintenance. According to the manual issued by the Ministry of Land, Infrastructure, and Transport (hereinafter called “MLIT”)[1], the benefit from flood control projects is defined as the sum of the expected value of annual reduced damage cost over the evaluation years, considering discount rate of each year.

Theoretically, annual reduced damage cost is actually a random variable; accordingly, the sum of annual reduced damage cost is also random variable.Therefore, the benefit defined aboveis equivalent to the expectedvalue of the sum of annual reduced damage cost over the evaluation years when neglecting discount rate. According to central limit theorem, actual reduced damage cost is normally distributed around the benefit calculated from the expected value when evaluation period is long enough. However, this is not guaranteed for an actual evaluation period due to the low frequency of flooding. In other words, it is highly uncertain whether actual reduced damage cost is distributed around its expected value, i.e. benefit calculated from the expected value does not necessarily occur frequently.These facts show that the above deterministic approach to decision making of flood control projects based on cost-benefit analysis includes the uncertainty of the probabilistic characteristics of reduced damage cost.If we can estimate the probability distribution of reduced damage gained over evaluation period, we can get more comprehensive information from the distribution in addition to the expected value, e.g. the probability with which actual reduced damage cost becomes larger than expected value. These will help advanced decision making of flood control projects based on multiple criteria.

In a literature, several uncertainties for benefit calculation have been discussed, e.g.the uncertainty of risk evaluation process including flood frequency analysis [3] and risk premium. For example, in the field of urban planning,it is discussed that willingness to pay becomes higher than the expected value of reduced damage cost because of risk aversion[4]. Risk premium is induced from the probability distribution of reduced damage cost;therefore, estimating the probability distribution of benefit is important to explicitly incorporate the idea of risk premiuminto in the process of decision making for disaster prevention investment.

On the other hand, recent studieshave developed methods forestimatingaflood risk curve, i.e. the probability distribution of annual maximum inundation damage cost, considering influential factors of risk evaluation such as rainfall time-space distribution and/or hydrograph variations[2],[5].For instance, Tanaka et.al.[5] showedthat considering therainfall time-space distribution makes it possible to enhance the accuracy of evaluating flood disaster risk. This study demonstratesthe derivation of the probability distribution of *T*-yearcumulative reduced-damage cost by generating hugenumber of rainfall events in consideration of rainfall time-space distribution.

From these backgrounds, the purpose of this study is estimating the distribution of *T*-yearcumulativereduced damage which is defined as sum of annual reduced damage cost over*T* years, and quantifying the probability with which we can actually benefit from flood control projects. The study area is Yodo River basin (8,240km^2) and several construction scenarios of the existing dams in the Kizu River basin (Takayama Dam, Murou Dam, Shorenji Dam, Hinachi Dam, Nunome Dam) are evaluated.For each project, we estimatethe probability distribution of T-yearcumulativereduced damage cost at conjunction area of three tributaries, Kizu River, Uji River, and Katsura River, and analyzeits stochastic characteristics. The impact of design of projects and evaluation period on the estimated distribution is also examined.

II. Basic Concept of Estimating Probability Distribution of TYear cumulativeReduced Damage Cost

According to the manual of cost-benefit analysis issued by MLIT[1], benefit of flood defenses over T years$B(T)$ is defined as

$$B(T) = \sum_{t=1}^{T} \frac{b}{(1+r)^{t-1}}, \tag{1}$$

where b is expected value of annual reduced damage; $r$is discount rate; and $T$is evaluation period. Denoting annual flooding damage before and after flood control investment as D_a and D_b, respectively,b is defined withannual reduced damage$D_{\mathrm{R}} = D_b - D_a$as

$$b = \mathrm{E}[D_{\mathrm{R}}] = \mathrm{E}[D_{\mathrm{b}}] - \mathrm{E}[D_{\mathrm{a}}]. \tag{2}$$

On the other hand, we define T-yearcumulativereduced damage cost $D_{\mathrm{R,all}}(T)$ as follows.

$$D_{\mathrm{R,all}}(T) = D_{\mathrm{b,all}}(T) - D_{\mathrm{a,all}}(T) \tag{3}$$

where $D_{\mathrm{b,all}}(T)$ and $D_{\mathrm{a,all}}(T)$ are cumulativedamage cost before and after investment, and denoted as

$$D_{\mathrm{b,all}}(T) = \sum_{t=1}^{T} D_{\mathrm{b},t}\ , D_{\mathrm{a,all}}(T) = \sum_{t=1}^{T} D_{\mathrm{a},t}, \tag{4}$$

respectively, where $D_{\mathrm{b},t}$ and $D_{\mathrm{a},t}$ are flooding damage costs of year $t$before and after flood control investment. In this study, we estimate probability distribution of T -yearcumulativereduced damage cost $D_{\mathrm{R,all}}(T)$ defined in(3). In addition, we investigate impact of evaluation period T length on the probability distribution of $D_{\mathrm{R,all}}(T)$. Note that expected value of$T$-yearcumulativereduced damage cost is obtained from(1) and(4), and we can confirm that benefit $B(T)$used in the manual [1] corresponds to the mean value of $D_{\mathrm{R,all}}(T)$ when neglecting discount rate, i.e.

$$\mathrm{E}\left[D_{\mathrm{R,all}}(T)\right] = \mathrm{E}[D_{\mathrm{b,all}}(T)] - \mathrm{E}[D_{\mathrm{a,all}}(T)] \\ = \sum_{t=1}^{T} \mathrm{E}\left[D_{\mathrm{b},t} - D_{\mathrm{a},t}\right] = \sum_{t=1}^{T} \mathrm{E}\left[D_{\mathrm{R},t}\right] = \sum_{t=1}^{T} b, \tag{5}$$

where $D_{\mathrm{R},t}$ is reduced damage cost of year$t$.

III. Methodology of Estimating Probability Distribution

In this research, we take a Monte Carlo approach to obtain enough number of samples of T-year cumulativereduced damage cost, by generatingenormous number of extreme rainfall events.Assume rainfall eventsfollow the following three assumptions which were used by Tanaka et.al.[5].

Assumption 1. When rainfall occurs, possible rainfall pattern is limited to the N patterns. Pattern ξ_i $(i = 1,2,\ldots,N)$ occurs with probability p_i. Denote rainfall duration time of pattern $\xi_i$as$d_{i.}$

Assumption 2. Basin averaged total rainfall r_{a}follows the conditional cumulativedistribution function (CDF)$G_{R_{\mathrm{a}}|D}(r_{\mathrm{a}}|d_i)$given $d_{i.}$

Assumption 3. Annual number of occurrences of rainfall eventsfollows the Poisson distribution withoccurrence ratio ofμ_{a}.

Define $r(x,y,t)$as rainfall intensity at a location (x,y) and time t, and rainfallpatternξ_i is defined as

$$r(x,y,t) = r_{\mathrm{a}}\xi(x,y,t), \qquad (x,y) \in A, 0 \le t \le d, \tag{6}$$

where $A$is the targeted watershed. $\xi(x,y,t)$ is normalized to satisfythe following equation:

$$\iint_A \left(\int_0^d \xi(x,y,t)dt\right) dxdy = S, \tag{7}$$

where$S$means the catchment area of targeted watershed. Rainfall duration time d is defined as follows.

$$d = \sup\{t | \xi(x,y,t) > 0,\ \exists(x,y) \in A, t > 0\}. \tag{8}$$

Definingthe starting time of rainfall as 0, $d$indicates the maximum time until which rainfall lasts at a certain point in the watershed.

Following these assumptions, we generate immense samples of T-yearcumulativereduced damage cost, following the steps shown below.

Step 1. Generate the annualnumber of occurrencesof rainfall events$n$from the Poisson distribution

Step 2. Prepare N past rainfall eventsand extractrainfall patterns $\xi_i(x,y,t)$ normalized by total rainfall amount. Note that $t$satisfies inequality, $0 \le t \le d_i, (x,y) \in A$ where $d_{i.}$ isthe rainfall duration time of pattern $\xi_i(x,y,t)$; $A$is the catchment area. Generate one pattern with the probability $1/N$, i.e.randomly.

Step 3. By following Step 2, the conditional CDF $G_{R_{\mathrm{a}}|D}(r_{\mathrm{a}}|d_i)$ corresponding with the generated rainfall pattern $\xi_i(x,y,t)$is determined. Generate total rainfall amount$r_{\mathrm{a}}$from the conditional CDF$G_{R_{\mathrm{a}}|D}(r_{\mathrm{a}}|d_i)$. From the obtained total rainfall amount r_{a} and its spatiotemporal pattern $\xi_i(x,y,t)$, one rainfall event is determined

Step 4. Running a rainfall-runoff model and an inundation model, calculate damage costs with/without investment, D_b and D_a, f.Then, calculate the reduced damage cost for the generated rainfall event.

Step 5. Repeating Steps 2 to 4 for$n$ times, sum reduced damage costs of each rainfall events and obtainone sample of annual reduced damage costD_{R}.

Step 6. Repeating Steps 1 to 5 for T times, sum annual reduced damageand get one sample of T-yearcumulativereduced damage cost$D_{\mathrm{R,all}}(T)$. In this research, discount ratewas eliminated to focus the discussion on probabilistic characteristics of reduced damage cost.

Step 7. Repeat Step6 for M times and estimate the probability distribution of T-yearcumulativereduced damage cost from the many samples of$D_{\mathrm{R,all}}(T)$.

IV. Application to the Dam Construction Projects

A. Rainfall generation and Damage cost calculation

The above presented method was applied to four different scenarios of dam construction projects for five actual damsalongthe Kizu River(Takayama Dam, Murou Dam, Shorenji Dam, Hinachi Dam, Nunome Dam), whose locations are displayed in fig. 1. According toTanaka et.al.[6], when flood beyond design level occurs, the river overflows around the three tributaries' conjunction area, and the overtopping probability of downstream Yodo River becomes very low. Hence we analyze the effect of dams from the reduced damage cost at three tributaries' conjunction area indicated by the red colored box in fig.1. Inthe flood simulation, we do not consider dyke breach, and only consider damage from overtopping.

We used the conditional CDF of basin-averaged rainfall on duration $G_{R_{\mathrm{a}}|D}(r_{\mathrm{a}}|d_i)$ on the Yodo River basin and rainfall-runoff/inundation models estimated/constructed by Tanaka.etal. [6] as follows. Rain gauge data for 35 years (1980 to 2014)at 156 stations were used in analysis. They extracted spatiotemporal rainfall patternsofthe 1,317 rainfall events observed over 35 years, i.e. $N = 1{,}317$. From the basin-averaged rainfall and duration time of the 1,317 rainfall events over 35 years, they estimated the conditional probability distribution of one basin-averaged rainfall on duration time. Onerainfall event is defined as a series ofrainfall eventswithout dry period longer than 5 hours. In order to estimate conditional probability distribution, the generalized Pareto distribution and two-variable exponential distribution were fitted to the marginal distribution of basin-averaged rainfall$G_{R_{\mathrm{a}}}(r_a)$ and rainfall duration time$G_D(d)$, respectively. Their correlation structure $\mathrm{C}(G_{R_{\mathrm{a}}}(r_a), G_D(d))$was modelled by using the normal copula. According to the Theorem of Sklar[7], this notation indicates the simultaneous distribution $G_{R_{\mathrm{a}},D}(r_a, d)$. Finally,the conditional cumulative distribution was obtained as

$$G_{R_{\mathrm{a}}|D}(r_{\mathrm{a}}|d) = \frac{1}{g_D(d)}\frac{\partial}{\partial d}G_{R_{\mathrm{a}},D}(r_a, d). \tag{9}$$

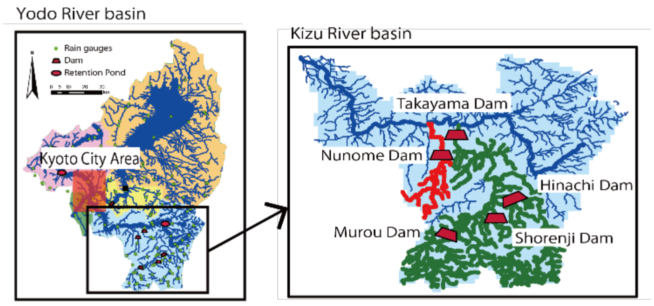

Fig. 1. Figure of Yodo River basin.

As a rainfall-runoff model used was1K-DHM[8], a distributed rainfall runoff model based on a kinematic wave flow approximation. 1K-DHM calculates the slope runoffand river routingat each cell at around1km (30 sec) resolution over a two dimensional domain. Slope runoffis simulated with the continuity and momentum equations of the kinematic wave theory,considering saturated and unsaturated subsurface flows and surface flow with the following discharge-storage relation.

$$q = \begin{cases} v_c d_c \left(\frac{h}{d_c}\right)^{\beta} \quad (0 \le h \le d_c) \\ v_c d_c + v_a(h - d_c) \ \ (d_c \le h \le d_a) \\ \alpha(h - d_c)^m + v_a(h - d_c) + v_c d_c (d_a \le h) \end{cases} \tag{10}$$

where,q is the slope runoff dischargeper unit width;h is the water depth; d_a and d_c are the water depth corresponding to the maximum water content ofsaturated and unsaturated soil layers, respectively; v_a and v_c are the water velocity ofsaturated and unsaturated soil layers, respectively; $\alpha = n/\sqrt{\sin\theta}$; n is the roughness coefficient; and $\theta$is the slope gradient. River flow is simulated with a kinematic wave model. All dams in Yodo basin are modeled in accordance with their operation rules, and rainfall-runoff before/afteraproject is representedby excluding/includingthe corresponding dam models.

Overflow inundation over the floodplain was modelled withone dimensional local inertial equation for river routing and two dimensional one for floodplain simulation. One dimensional local inertial equation is denoted as

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0, \tag{11}$$

$$\frac{\partial Q}{\partial t} + gA\frac{\partial (h + z)}{\partial x} + gn^2\frac{Q|Q|}{R^{4/3}A} = 0, \tag{12}$$

where$t$ is the time; x is the location in the downstream direction; A is the cross-sectional area; Q is the discharge; z is the elevation; g is the gravity acceleration; n is the roughness coefficient; and R is the hydraulic radius. Neglecting the advective term ofthe Saint-Venant equation, high speed arithmetic operation is realizedwith the same level of accuracy asthe diffusion equation. Damage cost is calculatedfor house, households, and office depreciable/stock assets, by multiplying exposed asset at each cell of the inundation model by damage ratio which is a function of maximum flood depth. The function is defined inthe manual of MLIT [1].

In accordance with MLIT's manual[1], evaluation period *T*was set as 50 years..When disaggregating a rainfall event into to basin-averaged cumulativerainfall and its spatiotemporal pattern as in this study, it is obvious that flood damage cost monotonically increases with basin-averaged rainfall for a fixed rainfall pattern; therefore, we have developed a relation between cumulativerainfall and resulting flood damage cost for all the rainfall patterns before Monte Carlo simulation, then in Step 4 to calculate flood damage cost, flood damage cost for the generated rainfall event was calculated by linearly interpolating the relation for the generated rainfall pattern,instead of running rainfall-runoff/inundation models for all the generated events. Repeating the calculation of *T*-yearcumulativereduced damage costfor $M = 100{,}000$ times, we estimatedthe probability distribution of the benefit of each scenario.

B. *Trial Calculation Results of 50* years *total reduced damage cost.*

Among the five dams, we first set project J: there is no dam on the Kizu Riverand construct Takayama Dam on the basin. Figure 2 shows the histogram of simulated *T*-yearcumulativereduced damage cost. Red line indicates the expected value. JPY is converted to USD at a rate of100yen per dollar.Obviously,the distribution is skewed leftwards with a long tail. The probability with which the benefit is equal to or over the expected value is about 10-20%, and one with whichbenefit is obtained is about 55%. On the other hand, extremely large benefit values much larger than the median value occurs with a certain probability. These indicated that the expected value integrated different types of benefit andmadeit difficult to grasp the whole characteristic of thebenefit of the project. By estimating the distribution of reduced damage cost asfig.2, more detailed information as demonstrated above becomes available, which is expected to support more rational decision making.

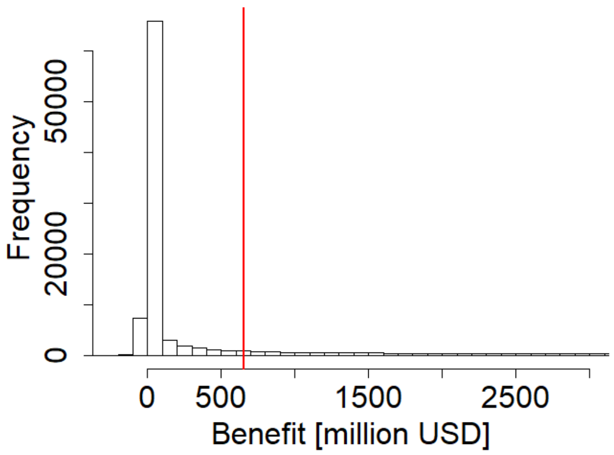

Fig. 2. Histgram of 50-yearscumulativereduced damage cost of the project constructing Takayam Dam. Red line indicates expected value.

V. DIFFERENCES OF PROBABILITY DISTRIBUTION BETWEEN PROJECTS

Among the five target dams, the Takayama Dam has the largest flood storage capacity. As shown in fig.1, amongthe other four dams, Nunome Dam is located on a different tributary river. To examine the impact of the location of dams on the estimated probability distribution, we splitthe construction project of the four dams intoprojects K, and project L:Project K first constructs three dams except for the Nunome Dam (Shorenji Dam, Murou Dam, and Hinachi Dam), and Project L then constructs the Nunome dam. Table.1 shows the statistics of three projects (J, K, L) gained from their probability distribution of 50-year cumulativereduced damage cost. Comparing with ProjectsK and L, Project L shows lower skewness and higher probability of getting more befit equal to or more thanexpected value, and the occurrence of benefit.

As shown in fig.1, in the Project K, TakayamaDam is already constructed in the same tributary, so dams constructed in Project K can mitigate damage only forflood causing inflow volume over the storage capacity of Takayama Dam. On the other hand, the Project L constructs Nunome Dam in the tributary where no dam has been constructed before; thus Nunome dammore frequently shows flood control effect than dams installed by Project K.Accordingly,Project L is expected to benefit the target area more frequently thanProject K. As shown in Table.1, difference between the expected valueof ProjectsK and L is about 26%.On the other hand,the probability with which the benefit equal to or more than its expected value by Project L is 230% larger than Project K, indicating that using other statistics provided different characteristics of projects from the expected value .

VI. RELATION BETWEEN PROBABILITY DISTRIBUTION AND CENTRAL LIMIT THEOREM

When evaluation period *T*is large enough, probability distribution of *T*-yearcumulativereduced damage cost theoretically converges to normal distribution, so reduced damage cost is assumed to distributearound its expected value. Figure.4 shows the variation of histogram of Project J when changing the evaluation period from 50 years to 1,000 years. For comparison of distribution characteristics,cumulativereduced damage cost was converged to annual

TABLE I. BENEFIT STATSITICS OF EACH PROJECT

Project	J	K	L
Expected value	655	197	157
Standard variarion	2240	804	355
Skewness	38.3	5.44	2.92
Probability with which benefit becomes larger than expected value	0.18	0.10	0.23
Occurrence probability of benefit	0.55	0.30	0.74

mean value by dividing evaluation year *T*. We can confirm that probability distribution of reduced damage cost skewed leftwards and have a long tail in limited evaluation period of 50 years, but the distribution approaches bilaterally symmetrical and the probability we can actually get benefit equal to or more than the expected valueincreases as the evaluation period becomes larger.This indicated that the general evaluation period of 50 years are not quite enough to represent the annual reduced damage cost by its expected value.

CONCLUSION

In the presentbenefit evaluation method, benefit is representedas the expectedvalue of cumulativereduced damage in evaluation period. However, flooding is a low-frequency event, especially in highly protected areas; thus,it is highly uncertain whether actual reduced damage cost is distributed around its expected value. In this research, in order to quantify stochastic characteristics of benefit, we developed an estimation method of *T*-yearcumulativereduced damage cost probability distribution, and applied the method to several scenarios of flood control projects on the Kizu River.

By estimating probability distribution of *T*-yearcumulativereduced damage cost, we can get more statistics as well as mean value, and reveal the characteristics of each project. Following findings were obtained in this study:

1. In all the four projects, probability distribution skewed leftwards, and the probability with which we can actually obtainthe benefit equal to or more than the expected valueis about 10-20%. On the other hand, extremely large benefit values is expected with a certain probability. In the process of averaging reduced damage cost,
2. The expected value of benefit and other statistics such as the probability with which benefit is obtained show different characteristics depending on projects
3. When we set longer evaluation period, 1000 years, distribution of cumulativereduced damage approaches normal distribution, and benefitwas evenly distributedaround its expected value, compared with offsetting evaluation period to 50 years. This indicated that *T*-yearcumulativereduced damage cost was distorted because the evaluation period was not long enough to represent the expected value as the benefit due to low frequency of flooding.

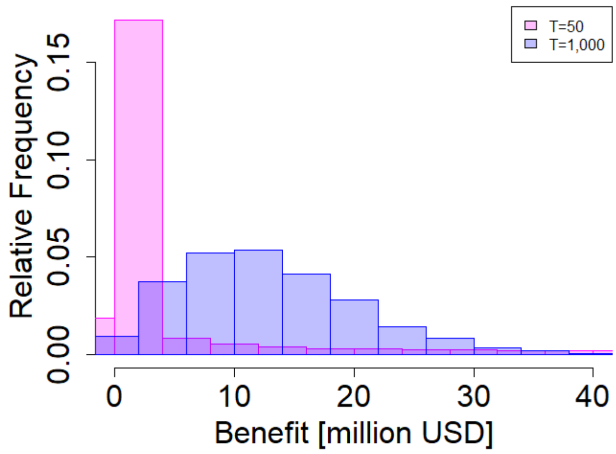

Fig. 3. Histgram of 50 and 1,000 years cumulativereduced damage cost of the project

This study proposed an idea to estimate the probability distribution of *T*-yearcumulativereduced damage cost for providing information about various statistics of reduced damage cost and making more rational decision making. We will discuss how to incorporate this idea into the actual decision making process for advanced decision making process of flood control projects.

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