

# Optimal reservoir operations under inflow scenarios in Nam Ngum River basin using Mixed-Integer Nonlinear Programming

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**Abstract**—Optimal reservoir operation is crucial, especially for large multipurpose reservoirs. The reservoir operation is complex and challenging subject to increasing water demands, variable reservoir inflow, and climate uncertainties. Nam Ngum 1 (NN1) is one of the key hydropower plants in the energy sector in Laos. This research aims to maximize the hydropower production under the impact of different reservoir inflow scenarios (observed inflow and inflow forecasting from time series model). Two time series models including Autoregressive (AR) and Autoregressive Integrated Moving Average (ARIMA) with different orders and parameters were used to simulate inflow. The simulated inflow obtained from the time series models was compared with the observed inflow. The best candidate model for each time series process was selected based on Akaike Information Criterion (AIC). ARIMA (2,1,3) was selected for monthly reservoir inflow simulation due to its ability to capture better low flow and high flow characteristics of the system. The NN1 reservoir operation optimization was formulated using Mixed Integer Non-linear Programming (MINLP) technique. The total annual and monthly power productions from optimization model under different inflow scenarios were analyzed. With optimized operation suggested by the model, the annual hydropower production can be increased 12.25% under observed inflow scenario and 2.22 % under simulated inflow scenario. The methodology for optimal operation demonstrated in this study can be used as a guideline for determining water release under different inflow scenarios in the Nam Ngum River basin and can be extended further for modelling multiple multipurpose reservoirs as a single system.

**Keywords**—Optimal reservoir operation, Inflow forecasting, Time series model, ARIMA, MINLP technique

## I. INTRODUCTION

Reservoir operation is one of the essential and challenging problems in water resources planning and management. Reservoir operation mainly involves determining the water

release to serve the purposes of the reservoir. One of the solutions to determine optimal water release can be derived using optimization techniques to solve the best solution for specific objective function [1]. To develop the optimal reservoir operations under limitation of available resources, Reference [2] demonstrated that defining the constrain, objective function and optimization technique is very important and challenging. The complexity of the problem deals with many complicated variables including reservoir inflow, volume of storage, water demand and water supply as well as climate uncertainty associated with the operational problem that must be addressed [3].

The Nam Ngum River basin (NNRB) in Lao PDR has faced several operational problems caused by natural variability such as the flood and drought and management of water supply such as lacking cooperation between stakeholders in the upper and lower stream. In addition, almost all the reservoirs in the NNRB have been operated based on outdated operation rule together with experience of the operators [4]. Traditionally, reservoir operations are based on heuristic procedures in which an optimal solution cannot be guaranteed because the operations sometime based on embracing rule curves and subjective judgments by the operator [5]. Furthermore, the uncertainty in reservoir operations such as change in climate and economic activities has complicated the operations. Reference [6] discussed that the conventional reservoir operation methods are often not adequate for establishing optimal operation decisions Reference [6] applied the Water Evaluation and Planning System (WEAP) model to simulate water distribution at the watershed scale, incorporating the climate change driven hydrologic cycle and water infrastructure operations. The results from study of [6] showed that the uncertainty affects reservoir inflow and water allocation. This confirms the complexity in reservoir operation. Many optimization and simulation models, such as dynamic programming (DP) model, system dynamic (SD) model, have

been developed and applied over the past several decades. However, the reservoir simulation method does not guarantee to yield the optimal operation because the operation is sometime based on idea and experience of engineers [7]. In a single reservoir operation, the operation is often required balancing among objectives including water supply reliability, hydropower generation, environmental flows, flood control, etc. [8]. To achieve the optimal multipurpose of reservoir operation, Reference [1] illustrated that it is a very difficult task due to conflicting multiple objectives, dimensionalities and nonlinearities [9].

According to the background and problems of reservoir operations, the optimization model possibly provides necessary information to improve operational water release for NNRB. Nam Ngum 1 (NN1) hydropower plant is chosen as a case study for this research. This study aims to optimize water release to meet domestic demand, irrigation demand and maximize the hydropower production. However, domestic demand and irrigation demand of NN1 are small, water release through the turbine is enough for them. Therefore, this research focuses only on determining water release that maximize hydropower production for NN1. The Mixed Integer Nonlinear Programming (MINLP) [10] technique used to optimize the reservoir operation under different inflow scenarios of forecasted inflow from time series model [11] and observed inflow would be investigated in this study. The framework and methodology for developing an optimal reservoir operation for the NN1 are presented in this study and expected to be extendable to other reservoirs in the NNRB.

## II. STUDY AREA

The Nam Ngum River is one of the main tributaries contributing to the Mekong River. The Nam Ngum River basin (see Fig. 1) is the fourth largest basins in Laos based on the area size.

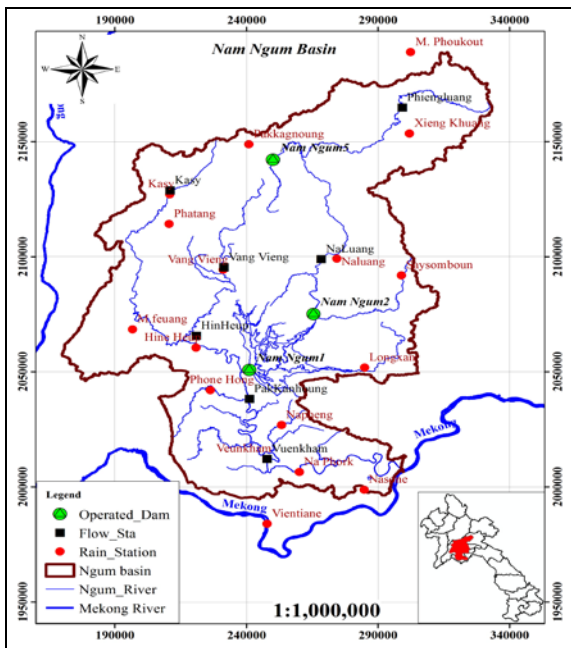


Fig. 1. Nam Ngum River basin system

The total area of the river basin is 16,931 km<sup>2</sup>, accounting for 7.3% of Laos. Major land use in the basin is forest covering approximately 81% of the entire basin area [4]. The NNRB is characterized by tropical climate with a distinct wet season and dry season. The annual average amount of rainfall over basin is 2,472 mm and ranges from more than 3,500 mm in the central region of basin near Vang Vieng district to below 1,400 mm in Xieng Khouang province in the north part of basin. The annual average outflow from the NNRB flow to Mekong River is 2.16 billion m<sup>3</sup> [4].

## III. RESERVOIR CONDITION

### A. Hydropower plant

The Government of Lao PDR has persistently projected to find the approach to obtain the maximum benefit from the hydropower generation, which is expected to increase significantly in the future, through the reservoir operations [12]. There are three existing cascade reservoirs on the main stream of the NNRB including the Nam Ngum 1 (NN1), Nam Ngum 2 (NN2), and Nam Ngum 5 (NN5) hydropower plants (see Fig. 1) used for hydropower generation.

The NN1 is a reservoir operated by EDL-Generation Public Company (EDL-GEN). It is located approximately 90 km north of Vientiane Capital. The main features of NN1 hydropower plant are summarized in TABLE I.

TABLE I. KEY FEATURES OF NN1 HYDROPOWER PLANT

Category	Data	Unit
Catchment Area	8,460	km <sup>2</sup>
Active Storage Capacity	4,700	MCM
Max. Flood Level	215	m
Full Supply Level (FSL)	212	m
Minimum supply Level (MSL)	196	m
Maximum Tailwater Level	178	m
Rated Flow per turbine	155	m <sup>3</sup> /s

### B. Reservoir operations

The decision on reservoir operations activity using rigid approach or rule often leads to ineffective energy production. Operation of reservoirs is a complex problem that involves many decision variables, multiple objectives as well as considerable risk and uncertainty. Reservoir operating rules provide general operation strategies for reservoir releases according to the current reservoir level, hydrological conditions, water demands and the time of the year. In order to obtain optimal operating rules, a large number of optimization and simulation models have been developed and applied.

### C. Description of data and tool

There are 14 years (2002–2015) of historical inflow time series data used in this study. The entire time series data was divided into two sets. The first set from 2002 to 2011 was used for calibration and the second set from 2012 to 2015 was used for validation. The simulated inflow time series were developed using time series models (AR and ARIMA).

Autoregressive model (AR) is a process developed to predict value of stationary data from autoregressive time series in previous time period [13]. The AR model can be given in following form:

$$Y_t(p) = \alpha_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + \varepsilon_t \quad (1)$$

A Moving Average model is similar to an Autoregressive model, except that instead of being a linear combination of past time series values, it is a linear combination of the past white noise terms [13]. The AR model can be given in following form:

$$Y_t(q) = \beta_0 + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_p \varepsilon_{t-p} + \varepsilon_t \quad (2)$$

ARIMA was developed by Box and Jenkins (1970) [13]. It can be used to forecast non-stationary time series because the behavior of the data recorded in the past is assumed to be sufficient to define pattern in the present and explain the trend of itself in the future. ARIMA model is the integration between AR and MA processes. The general formulation of ARIMA model is represented as:

$$Y_t(p, d, q) = \alpha_0 + \sum_{i=1}^p \phi_i \Delta Y_{t-p} + \sum_{j=1}^q \theta_j \varepsilon_{t-q} + \varepsilon_t \quad (3)$$

where:  $Y$  is variable at time  $t$ ,  $\alpha$  and  $\beta$  are constant for AR and MA,  $\phi$  and  $\theta$  are coefficients for AR and MA,  $p$  and  $q$  are the number of lagged forecast errors in the prediction equation for AR and MA,  $d$  is the number of non-seasonal differences needed for stationarity,  $\Delta$  is difference operator and  $\varepsilon$  is error.

Optimal operation model for the reservoir system was developed using mixed integer nonlinear programming (MINLP) which is solved using LINDOGlobal solver through General Algebraic Modelling (GAMS) language. The description and details of the GAMS model is omitted here and referred to [14] for more details.

According to the study of [15], the MINLP is referred to mathematical programming with continuous and discrete variables and nonlinearities in the objective function and constraints. Reference [16] also explained that MINLP problems are difficult to solve, because they combine all the difficulties of both of their subclasses: the combinatorial nature of mixed integer programs (MIP) and the difficulty in solving nonlinear programs (NLP). The MINLP can be written in following forms:

$$\begin{aligned} & \text{Maximize} && f(x, y) \\ & \text{Subject to} && g(x, y) > 0 \\ & && L \leq x, y \leq U \end{aligned} \quad (4)$$

where:  $f(x, y)$  is nonlinear objective function  
 $g(x, y)$  is nonlinear constraint function  
 $x, y$  are the decision variables  
 $L, U$  are Lower and Upper bounds

The time series of hydrological observations data used in this study were supported from Department of Meteorology and Hydrology (DMH), Lao PDR. The total data requirements for develop optimization model are listed below:

- Inflow to the reservoirs
- Water release through spillway
- Water release through turbine
- Reservoir characteristics (see TABLE I)
- Evaporation rate
- Surface area – Volume – Elevation Curve
- Observed power generation.

## IV. METHODOLOGY

### A. Inflow forecasting

Time series models were applied for simulating reservoir inflow. One time step forward forecasting technique is used. The results of average monthly inflow from AR and ARIMA models are compared with the observed inflow. The procedure of developing time series model is shown in Fig. 2 which involves four main steps of 1) collect data and perform initial check through visualization, 2) stationarizes the time series data, 3) estimate the time series model order and parameters and 4) develop time series model and simulate inflow.

The time series models were built using MATLAB toolbox. The simulated inflows from AR and ARIMA time series models were compared with observed inflow. The accuracy of the time series models was evaluated using Coefficient of Determination ( $R^2$ ), Root Mean Square Error (RMSE), and Mean Absolute Error (MAE). Selected inflow time series simulated from the model with relatively higher accuracy was then input to optimization model.

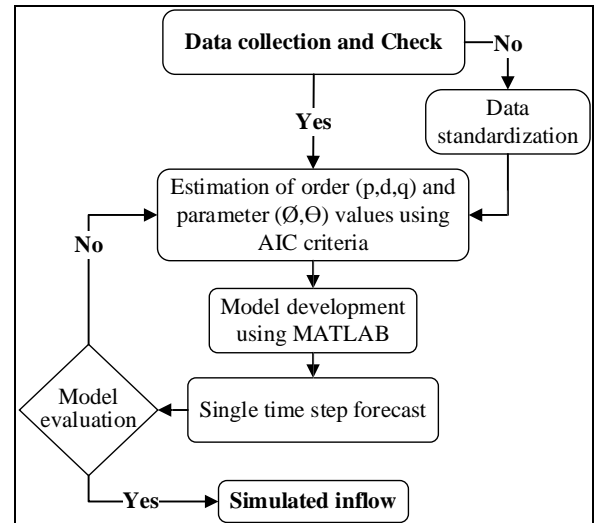


Fig. 2. Schematic of time series model

### B. Reservoir operations

Reservoir operations of NN1 is optimized under two different inflow scenarios. First scenario is optimized under observed inflow. Second scenario is optimized under simulated inflow from ARIMA. The procedure of developing the optimal reservoir operation model is illustrated in the Fig. 3.

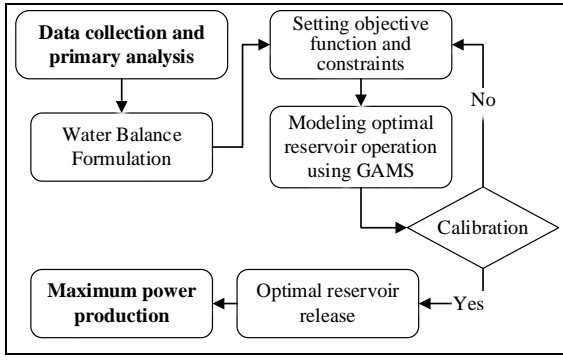


Fig. 3. Schematic of the optimal reservoir operations.

### 1) Reservoir water balance

The water balance equation is used to define water inflow and outflow of each reservoir. In this study, seepage is considered negligible and omitted from the water balance equation. Based on cascade reservoir system (series reservoir), the water balance equation is defined in the following form.

$$S_{t+1} = S_t + In_t - E_t - R_t - Spill_t \quad (5)$$

where:  $S_{t+1}$  is final storage at time  $t$ ,  $S_t$  is initial storage at time  $t$ ,  $In_t$  is reservoir inflow at time  $t$ ,  $E_t$  is evaporation loss at time  $t$ ,  $R_t$  is water release from reservoir through turbine at time  $t$ , and  $Spill_t$  is water over spillway at time  $t$ .

### 2) Objective function

Hydropower generation is of high importance to Laos due to its positioning to be the battery of Asia. The goal of hydropower generation is to achieve domestic hydropower demand and to boost economic growth. Therefore, maximizing the monthly power generation capacity is treated as the objective function in this study. Mathematically the objective function is given by:

$$\text{Maximize: } P_{Total} = \sum_{t=1}^n (\eta \times \gamma \times R_t \times H_t \times T) \quad (6)$$

$$n = 1, 2, 3, \dots$$

where  $P_{Total}$  is total electricity production (MWh),  $\eta$  is dimensionless efficiency of the turbine,  $\gamma$  is specific weight of water ( $\approx 9.81 \text{ KN/m}^3$ ),  $R_t$  is water release through the turbine ( $\text{m}^3/\text{s}$ ),  $H_t$  is height difference between inlet and outlet (m) and  $T$  is time for generate power in a day (hr).

### 3) Constraints

The propose of mathematical optimization model for NN1 reservoir considers constraints which involve release through turbine and spillway, storage volume and river capacity. The hydrologic, physical and operational characteristics of the reservoir are also considered.

## V. RESULT AND DISCUSSION

The study was aimed to optimize water release for maximizing hydropower production using MINLP technique under observed inflow and simulated inflow from time series model scenarios. The results of the developed MINLP under two different inflow scenarios were discussed in the following section.

### A. Inflow prediction

#### 1) Model selection

For the calibration data sets (2002 - 2011), the performance of 11 candidate time series models including AR(1) to AR(5) and ARIMA(1,1,1) to ARIMA(4,1,3) were assessed based on the AIC value as shown in TABLE II. The best candidate model from AR and ARIMA processes was selected based on the minimum value of AIC. The AR (4) and ARIMA (2,1,3) models with the AIC value of 189.555 and 183.810, respectively were selected.

TABLE II. AIC VALUES FOR TIME SERIES MODEL

No.	Model candidate	AIC value
1	AR(1)	245.120
2	AR(2)	195.878
3	AR(3)	194.441
<b>4</b>	<b>AR(4)</b>	<b>189.555</b>
5	AR(5)	196.421
6	ARIMA(1,1,1)	235.881
7	ARIMA(2,1,1)	198.212
8	ARIMA(2,1,2)	228.304
<b>9</b>	<b>ARIMA(2,1,3)</b>	<b>183.810</b>
10	ARIMA(3,1,3)	196.261
11	ARIMA(4,1,3)	211.426

The difference of AIC value between AR (4) and ARIMA (2,1,3) models is very small. Therefore, both AR (4) and ARIMA (2,1,3) models are tentatively selected for monthly reservoir inflow forecasting for NN1. The parameter values of the AR (4) and ARIMA (2,1,3) models shown in TABLE III. suggest that the first order of AR and ARIMA is more significant than the other orders. This means that the events of short lag time have higher correlation than the events of long lag time.

TABLE III. PARAMETER ESTIMATES

Parameter	AR(4)	ARIMA(2,1,3)
$\phi_1$	0.9159	1.4322
$\phi_2$	0.0197	-0.6505
$\phi_3$	-0.2161	-0.6821
$\phi_4$	-0.0576	-
$\phi_5$	-0.0082	-
$\theta_1$	-	-0.1053
$\theta_2$	-	-0.154
$\theta_3$	-	-0.3223
$\theta_4$	-	-0.5216

#### 2) Inflow forecasting

The simulated inflow is validated using the independent data set (2012-2015) and the results are shown in Fig 4 and Fig 5. In the Fig 4, it is clear that the AR (4) model can acceptably

capture the overall pattern of inflow but not high flow events. The AR (4) underestimates peak and detects peak too early. When compared to AR (4), the ARIMA (2,1,3) model can capture better low flow and high flow characteristics (see Fig 5). The performance of the time series models based on RMSE, R2 and MAE, are shown in TABLE IV.

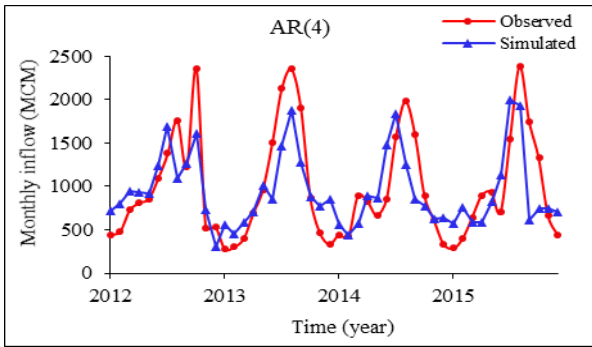


Fig. 4. Comparison between inflow from AR (4) and observed inflow

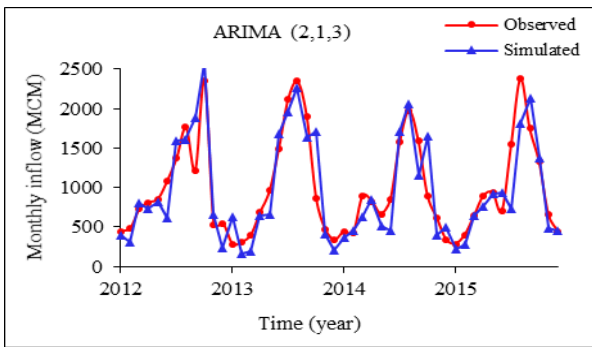


Fig. 5. Comparison between inflow from ARIMA (2,1,3) and observed inflow

TABLE IV. STATISTICAL PERFORMANCE INDICES OF TIME SERIES MODEL

Model	R <sup>2</sup>	RMSE (MCM)	MAE (MCM)
AR(4)	0.54	154.26	121.45
ARIMA(2,1,3)	0.78	115.41	84.27

From TABLE IV, it is clear that the ARIMA (2,1,3) model outperformed AR (4) in terms of all performance indices. According to the better performance, the simulated inflow from ARIMA (2,1,3) will be used as input to MINLP. The results of optimal reservoir operations under two different inflow scenarios including observed inflow and ARIMA inflow are discussed in the following section.

### B. Optimal reservoir operations under different inflow scenarios

The results of monthly and annual hydropower productions under different inflow scenarios were analyzed in the following section.

#### 1) Monthly hydropower production

The optimal average monthly power production under the different inflow scenarios compared with the actual production

is shown in Fig 6. It shows that the average monthly power production varies from 58,310 MW to 151,165 MW for the ARIMA inflow. The average monthly power production ranging from 78,886 MW to 120,592 MW was also obtained from observed inflow. In all periods, the optimum hydropower under observed inflow scenario (Opt\_power (Obs\_inflow)) were greater than that of the ARIMA scenario (Opt\_power (ARIMA\_inflow)) especially, for wet season (May - Oct). This maybe because during the wet season (Jun - Sep), the reservoir was filled, and the optimal reservoir operations model kept the maximum hydropower production up to October. This difference demonstrated that the inflow forecasting from time series model affected to the reservoir operations. However, the power production in dry months of December through April slightly decreased compared with the observed power production.

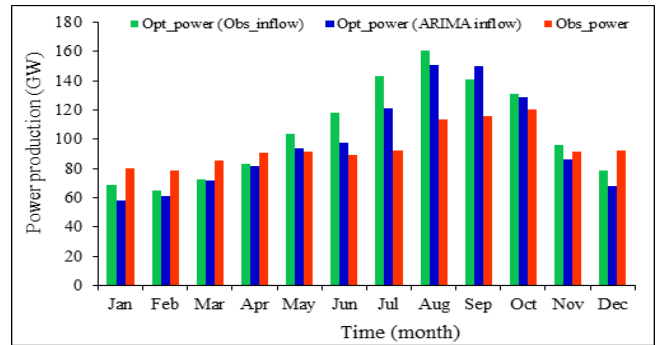


Fig. 6. Monthly hydropower production from MINLP compared with observed data.

#### 2) Annual hydropower production

The results of optimal annual hydropower production under the different inflow scenarios are shown in TABLE V and Fig. 7.

TABLE V. ANNUAL HYDROPOWER PRODUCTION SIMULATED BY MINLP

Year	Observed power (MWh)	Optimized power (MWh)	
		ARIMA_inflow	Obs_inflow
2012	1,040,707	1,312,074	1,230,977
2013	1,180,196	1,218,496	1,312,170
2014	1,141,816	1,185,843	1,324,773
2015	1,209,683	1,159,911	1,230,811
Average increase (%)		<b>2.22</b>	<b>12.25</b>

The result in TABLE V shows that the annual hydropower production optimized under the two inflow scenarios is higher than the observed power. The increase of 12.25% in power obtained from optimization under observed inflow scenario suggests that the application of the optimization model proposed here can contribute to better control of release. However, it needs to be noted that this increase may be because all the monthly inflow were input into the optimization model, while in the observed (actual) operation case, the operator has the inflow records only up until a previous month. For the optimization under ARIMA inflow, the hydropower production was slightly increased up to 2.22 % when compared with the observed power. However, the hydropower production was less than the optimization under observed inflow case because the

simulated inflow from ARIMA were generally lower than the observed inflow (see Fig. 5).

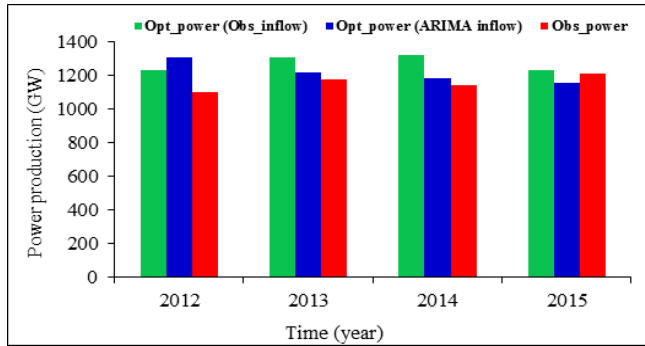


Fig. 7. Annual hydropower production from MINLP compared with observed data

## VI. CONCLUSION AND FUTURE STUDY

In this study the NN1 operations were optimized for maximizing the hydropower production through MINLP technique under two different reservoir inflow scenarios. In the first scenario, the hydropower was optimized under the observed inflow and in the second scenario, the hydropower was optimized under ARIMA simulated inflow. The optimization model was developed and solved using LINDOGlobal solver through GAMS software. The entire series of data for NN1 reservoir inflow of 14 years (2002 - 2015) was divided in to two sets for time series model: calibration (2002 - 2011) and validation (2012 - 2015). The time series model, ARIMA (2,1,3) was selected for simulating monthly reservoir inflow because it outperformed other time series model in capturing low flow and high flow characteristics and yielded the best values for all statistical indices used in this study.

The optimization model was found to increase the hydropower production in both inflow scenarios. However, in some months the optimum hydropower generated under the observed inflow scenario was greater than that of the ARIMA (2,1,3) scenario. Optimization under both inflow scenarios improved existing reservoir operation as an increase in hydropower generation was achieved. When comparing between the two inflow scenarios, the optimization under observed inflow scenario yielded higher increase in hydropower compared to that of the ARIMA inflow.

Research for simulating inflow with uncertainty e.g. from climate change should be investigated further to better capture future conditions. For the optimal reservoir operation model, it is needed to improve objective functions and constraints to more explicitly address the flood at the downstream. The initial results obtained from this study should be extended to be applied for multiple multipurpose reservoir operations.

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